Modeling And Performance Analysis Of BitTorrent-like Peer-to-Peer Networks

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Brief

- Fluid model
- Steady-state performance
- Effectiveness
- Local stability
- Free-riding
- Results

Terms

$x(t)$ Number of downloaders at time t

$y(t)$ Number of seeds at time t

λ Arrival rate of new requests. (Assumption: 1 downloader makes only 1 request)

μ Upload bandwidth. (Assumption: All peers have same μ)

c Download bandwidth. Note that $c \geq \mu$. (Assumption: All peers have same c)

θ Abort rate of downloaders.

γ Abort rate of seeds.

η Effectiveness. Takes values in $[0, 1]$.

Uploading Rate

If there is no constraint on c, the *total uploading rate* of the system is given by:

$$= \mu(\text{upload from } x(t) + \text{upload from } y(t))$$

$$= \mu(\eta x(t) + y(t))$$

If there is a constraint on c, then the total uploading rate is given by: $\min\{c\,x(t),\ \mu(\eta x(t) + y(t))\}$

Departure Rate

Rate of departures of $x(t)$ is given by:

$$= \text{Rate of } x(t) \text{ becoming } y(t) + \theta x(t)$$

$$= \min\{cx(t), \mu(\eta x(t) + y(t))\} + \theta x(t)$$

Fluid Model

A deterministic fluid model for evolution of $x(t)$ is given by:

$$\frac{dx}{dt} = x(t) \text{ who arrive} - x(t) \text{ who depart}$$

$$= \lambda - (x(t) \text{ who abort} + x(t) \text{ who become } y(t))$$

$$\frac{dx}{dt} = \lambda - \theta x(t) - \min\{cx(t), \mu(\eta x(t) + y(t))\}$$

Fluid Model …

A deterministic fluid model for evolution of $y(t)$ is given by:

$$\frac{dy}{dt} = y(t) \text{ who arrive} - y(t) \text{ who depart}$$

$$= x(t) \text{ who become } y(t) - \gamma y(t)$$

$$\frac{dy}{dt} = \min\{cx(t), \mu(\eta x(t) + y(t))\} - \gamma y(t)$$

Steady-State Performance

At steady state:

$$\frac{dx(t)}{dt} = \frac{dy(t)}{dt} = 0$$

Using $\bar{x}$ and $\bar{y}$ as *equilibrium values* of $x(t)$ and $y(t)$ in the fluid model equations, we get linear equations:

$$0 = \lambda - \theta \bar{x} - \min\{c\bar{x}, \mu(\eta\bar{x} + \bar{y})\}$$

$$0 = \min\{c\bar{x}, \mu(\eta\bar{x} + \bar{y})\} - \gamma\bar{y}$$

Case 1

When $\eta > 0$ and c is the constraint. Then:

$$c\bar{x} \leq \mu(\eta\bar{x} + \bar{y}) \Leftrightarrow \frac{1}{c} \geq \frac{1}{\eta}(\frac{1}{\mu} - \frac{1}{\gamma})$$

and

$$\min\{c\bar{x}, \mu(\eta\bar{x} + \bar{y})\} = c\bar{x}$$

Solving the steady state equations we get:

$$\bar{x} = \frac{\lambda}{c(1+\frac{\theta}{c})} \text{ and } \bar{y} = \frac{\lambda}{\gamma(1+\frac{\theta}{c})}$$

Case 2

$$\text{Let } \frac{1}{\nu} = \frac{1}{\eta}\left(\frac{1}{\mu} - \frac{1}{\gamma}\right)$$

When $\eta > 0$ and μ is the constraint. Then:

$$c\,\bar{x} \geq \mu(\eta\bar{x} + \bar{y}) \Leftrightarrow \frac{1}{c} \leq \frac{1}{\eta}\left(\frac{1}{\mu} - \frac{1}{\gamma}\right) \Leftrightarrow \frac{1}{c} \leq \frac{1}{\nu}$$

Solving the steady state equations we get:

$$\bar{x} = \frac{\lambda}{\nu(1 + \frac{\theta}{\nu})} \text{ and } \bar{y} = \frac{\lambda}{\gamma(1 + \frac{\theta}{\nu})}$$

Case 1 + Case 2

Define $\frac{1}{\beta} = \max\{\frac{1}{c}, \frac{1}{\eta}(\frac{1}{\mu} - \frac{1}{\gamma})\}$, then case 1 and 2 can be combined to

$$\bar{x} = \frac{\lambda}{\beta(1+\frac{\theta}{\beta})} \text{ and } \bar{y} = \frac{\lambda}{\gamma(1+\frac{\theta}{\beta})}$$

Avg. Download Time

Applying Little's Law at steady state:

$$\bar{x} \text{ that complete downloads} = \text{Rate of download completion} \times \text{Download time}$$

$$\text{Fraction of } \bar{x} \text{ who become seeds} = (\lambda - \theta\,\bar{x}\,)T$$

$$\frac{\lambda - \theta\,\bar{x}}{\lambda}\bar{x} = (\lambda - \theta\,\bar{x}\,)T$$

$$\text{Hence, } T = \frac{1}{\theta + \beta}$$

Inferences From T

- T is not related to λ. Hence, BT scales well. (Not really true.)
- More η, less T.
- More γ, more T.
- Initially when c increases, T decreases. (Good.) But once c isn't bottleneck, it doesn't affect T. Same for μ.

Effectiveness

$$k = \min \{x - 1, K\}$$

$$\eta = 1 - \mathbb{P}\{x_i \text{ has no piece that other } x_i \text{ need}\}$$

Effectiveness …

$$\eta = 1 - \mathbb{P}\{x_j \text{ needs no piece from } x_i\}^k$$

$$\eta = 1 - \mathbb{P}\{x_j \text{ and } x_i \text{ have the same pieces}\}^k$$

$$\eta \approx 1 - (\frac{\log N}{N})^k$$

Local Stability

From Case 2

$$A = \begin{pmatrix} -(\mu\eta + \theta) & -\mu \\ \mu\eta & -(\gamma - \mu) \end{pmatrix}$$

roots (ψ) are negative real.

Free-Riding

$$= \text{number of uploaders to } j \times \text{upload rate per peer per connection}$$

$$= \left(N \frac{1}{N - n_u}\right) \times \left(\frac{\mu}{n_u + 1}\right)$$

$$\text{For large } N \approx \frac{\mu}{n_u + 1}$$

Results

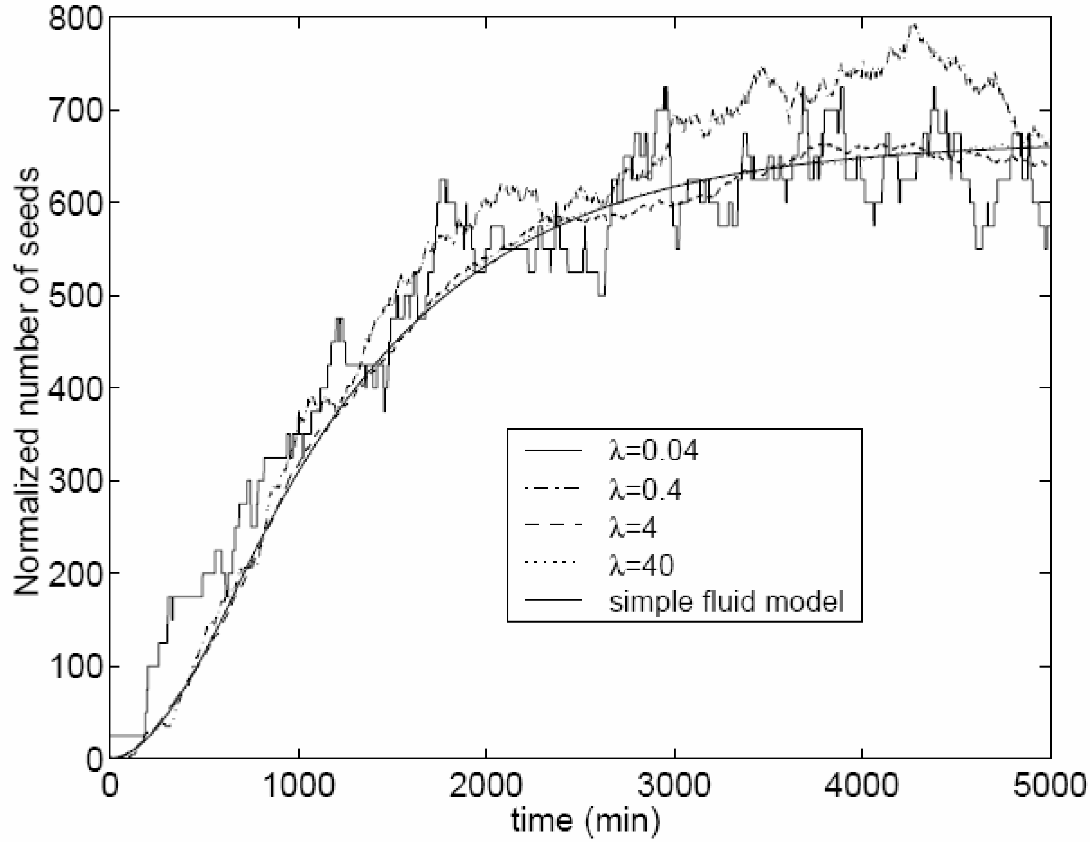

Figure 1: Experiment 1 : The evolution of the number of seeds as a function of time

Results …

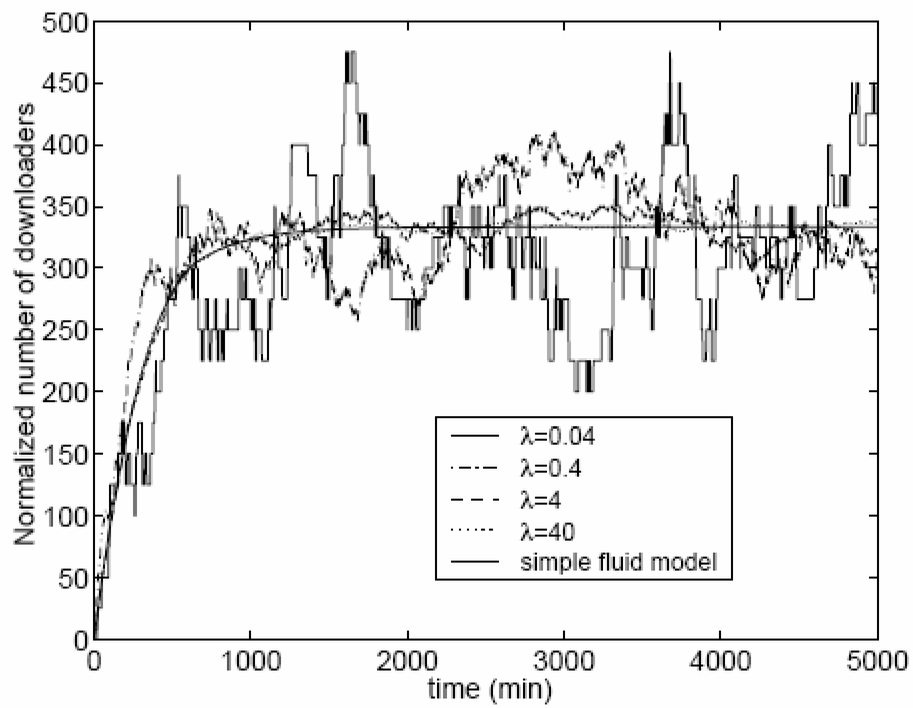

Figure 2: Experiment 1 : The evolution of the number of downloaders as a function of time

Results

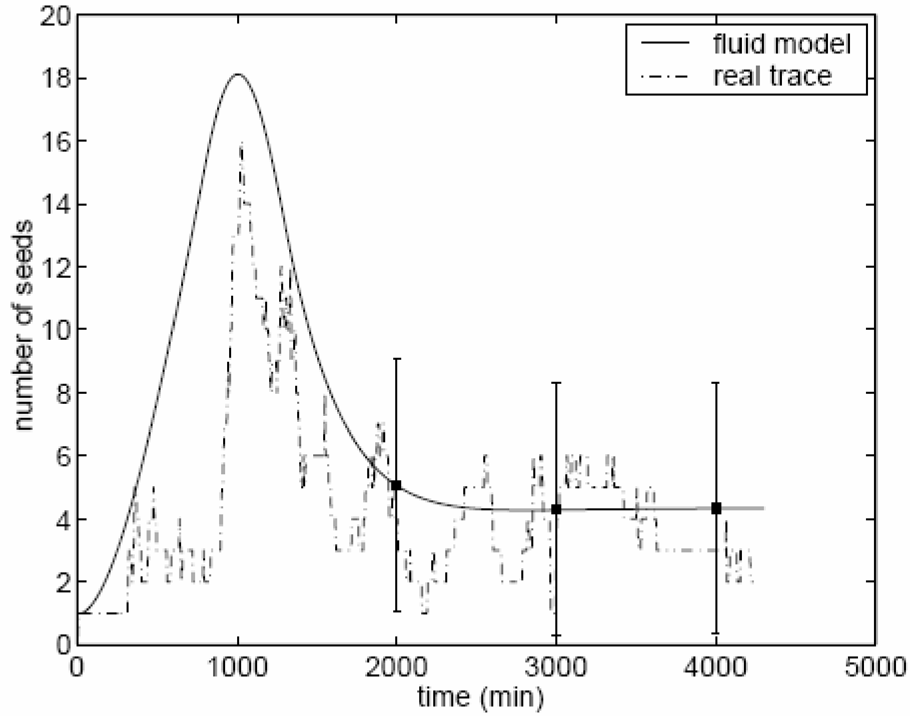

Figure 7: Experiment 3 : Evolution of the number of seeds

Results …

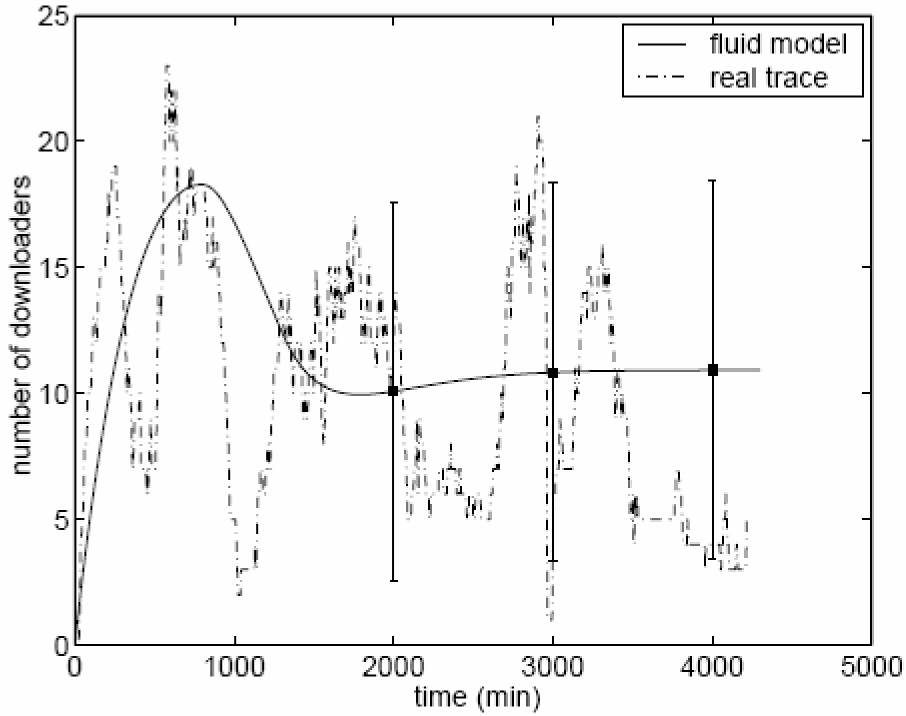

Figure 8: Experiment 3 : Evolution of the number of downloaders

Questions???

Thank You!