CS6282: Modeling And Performance Analysis Of BitTorrent-Like Peer-To-Peer Networks

(Notes from the paper)

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1. Terms

$x(t)$ Number of downloaders at time t

$y(t)$ Number of seeds at time t

λ Arrival rate of new requests. (Assumption: 1 downloader makes only 1 request)

μ Upload bandwidth. (Assumption: All peers have same μ)

c Download bandwidth. Note that $c \geq \mu$. (Assumption: All peers have same c)

θ Abort rate of downloaders.

γ Abort rate of seeds.

η Effectiveness. Takes values in $[0, 1]$.

2. Fluid Model

If there is no constraint on c, the *total uploading rate* of the system is given by:

$$= \mu(\text{upload from } x(t) + \text{upload from } y(t))$$

$$= \mu(\eta x(t) + y(t))$$

If there is a constraint on c, then the total uploading rate is given by: $\min\{cx(t),\ \mu(\eta x(t) + y(t))\}$

Rate of departures of $x(t)$ is given by:

$$= \text{Rate of } x(t) \text{ becoming } y(t) + \theta x(t)$$

$$= \min\{cx(t), \mu(\eta x(t) + y(t))\} + \theta x(t)$$

A deterministic fluid model for evolution of $x(t)$ is given by:

$$\frac{dx}{dt} = x(t) \text{ who arrive} - x(t) \text{ who depart}$$

$$= \lambda - (x(t) \text{ who abort} + x(t) \text{ who become } y(t))$$

$$\frac{dx}{dt} = \lambda - \theta x(t) - \min\{cx(t), \mu(\eta x(t) + y(t))\}$$

A deterministic fluid model for evolution of $y(t)$ is given by:

$$\frac{dy}{dt} = y(t)\,\text{who arrive} - y(t)\,\text{who depart}$$

$$= x(t)\,\text{who become}\,y(t) - \gamma y(t)$$

$$\frac{dy}{dt} = \min\{cx(t), \mu(\eta x(t) + y(t))\} - \gamma y(t)$$

3. Steady-State Performance

At steady state:

$$\frac{dx(t)}{dt} = \frac{dy(t)}{dt} = 0$$

Using $\bar{x}$ and $\bar{y}$ as *equilibrium values* of $x(t)$ and $y(t)$ in the fluid model equations, we get these linear equations:

$$0 = \lambda - \theta\bar{x} - \min\{c\bar{x}, \mu(\eta\bar{x} + \bar{y})\}$$

$$0 = \min\{c\bar{x}, \mu(\eta\bar{x} + \bar{y})\} - \gamma\bar{y}$$

Case 1:

When $\eta > 0$ and c is the constraint. Then:

$$c\bar{x} \le \mu(\eta\bar{x} + \bar{y}) \Leftrightarrow \frac{1}{c} \ge \frac{1}{\eta}(\frac{1}{\mu} - \frac{1}{\gamma})$$

and

$$\min\{c\bar{x}, \mu(\eta\bar{x} + \bar{y})\} = c\bar{x}$$

Solving the steady state equations we get:

$$\bar{x} = \frac{\lambda}{c(1 + \frac{\theta}{c})}\ \text{and}\ \bar{y} = \frac{\lambda}{\gamma(1 + \frac{\theta}{c})}$$

Case 2:

$$\text{Let}\ \frac{1}{\nu} = \frac{1}{\eta}(\frac{1}{\mu} - \frac{1}{\gamma})$$

When $\eta > 0$ and μ is the constraint. Then:

$$c\bar{x} \ge \mu(\eta\bar{x} + \bar{y}) \Leftrightarrow \frac{1}{c} \le \frac{1}{\eta}(\frac{1}{\mu} - \frac{1}{\gamma}) \Leftrightarrow \frac{1}{c} \le \frac{1}{\nu}$$

Solving the steady state equations we get:

$$\bar{x} = \frac{\lambda}{\nu(1 + \frac{\theta}{\nu})}\ \text{and}\ \bar{y} = \frac{\lambda}{\gamma(1 + \frac{\theta}{\nu})}$$

Case 1 + Case 2:

Define $\frac{1}{\beta} = \max\{\frac{1}{c}, \frac{1}{\eta}(\frac{1}{\mu} - \frac{1}{\gamma})\}$, then case 1 and 2 can be combined to get:

$$\bar{x} = \frac{\lambda}{\beta(1 + \frac{\theta}{\beta})} \text{ and } \bar{y} = \frac{\lambda}{\gamma(1 + \frac{\theta}{\beta})}$$

Applying Little's Law at steady state:

$$\bar{x} \text{ that complete downloads} = \text{Rate of download completion} \times \text{Download time}$$

$$\text{Fraction of } \bar{x} \text{ who become seeds} = (\lambda - \theta\,\bar{x}\,)T$$

$$\frac{\lambda - \theta\,\bar{x}}{\lambda}\bar{x} = (\lambda - \theta\,\bar{x}\,)T$$

$$\text{Hence, } T = \frac{1}{\theta + \beta}$$

What can we infer?

- T is not related to λ. Hence, BT scales well. (Not really true.)
- More η, less T.
- More γ, more T.
- Initially when c increases, T decreases. (Good.) But once c isn't bottleneck, it doesn't affect T. Same for μ.

Case 3:

When $\eta = 0$ (download only from y), if $\gamma > \mu$, the system dies quickly since seeds die off.

4. Effectiveness

If, K is the max number of peers that can be connected to a downloader, and k is the number of peers x_i is connected to, then $k = \min\{x - 1, K\}$

$\eta = 1 = \text{every } x_i \text{ is connected and downloading from other } x_i = \text{every } x_i \text{ is connected to } a\,x_i \text{ which has } a \text{ piece it needs}$

$$\eta = 1 - \mathbb{P}\{x_i \text{ has no piece that other } x_i \text{ need}\}$$

Assuming piece distributions are independent and identical and N is number of pieces,

$$\eta = 1 - \mathbb{P}\{x_j \text{ needs no piece from } x_i\}^k$$

$$\eta = 1 - \mathbb{P}\{x_j \text{ and } x_i \text{ have the same pieces}\}^k$$

$$\eta \approx 1 - (\frac{\log N}{N})^k$$

What can we infer?

- Bigger files download better (large N)
- Even with small k, good η is obtained.

5. Local Stability

From Case 2 linear equations for $\frac{dx(t)}{dt}$ and $\frac{dy(t)}{dt}$ we get the coefficient matrix :

$$A = \begin{pmatrix} -(\mu\eta+\theta) & -\mu \\ \mu\eta & -(\gamma-\mu) \end{pmatrix}$$

Eigenvalues of A determine stability of $\bar{x}$ and $\bar{y}$. If ψ is eigenvalue of A_1 then, $\det(\psi I - A) = 0$. This gives the characteristic equation (9) which is of the form $\mathcal{A}x^2 + \mathcal{B}x + \mathcal{C} = 0$.

We've seen that $\eta > 0$, the above $\mathcal{B} > 0$ and $\mathcal{C} > 0$. This means roots (ψ) are negative real.

From Handout 8, we see that it means that the linear system of equations of Case 2 is stable.

Similarly for Case 1.

Hence, BT system is locally stable.

6. Free-Riding

In BT, each peer can have (n_u uploads + 1 optimistic upload).

Since free-rider j doesn't contribute to anyone, he can only squeeze into the 1 optimistic upload of each peer.

A peer might connect to j during $\frac{1}{N - n_u}$ of the time.

So, download rate of j is:

$$= \text{number of uploaders to } j \times \text{upload rate per peer per connection}$$

$$= (N\frac{1}{N - n_u}) \times (\frac{\mu}{n_u + 1})$$

$$\text{For large } N \approx \frac{\mu}{n_u + 1}$$

So, free-riders get a ride, but not so free.

7. Results

See graphs.

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