Rigid Body And Fluid Interplay Solver

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Outline

Introduction

Implementation based on:

- *Rigid Fluid* by Mark Carlson et al. (SIGGRAPH '04)
- Fluid dynamics solver by Jos Stam
  - *Stable Fluids* (SIGGRAPH '99)
  - *Real-Time Fluid Dynamics For Games* (GDC '03)

Domains

- $C = F \cup R$
- C : Computational domain
- F : Fluid domain
- R : Rigid body domain

Governing Equations

$u^n \longrightarrow u^{n+1}$

1. $u^n \longrightarrow u^*$: Solve Navier-Stokes Equations.
2. $u^* \longrightarrow \hat{u}$: Collision and relative density forces.
3. $\hat{u} \longrightarrow u^{n+1}$: Rigid body motion.

Rigid Body Velocity

In rigid body domain (R), the rigid body velocity ($\hat{u}_R$) is composed of :

- Translational velocity ($\hat{v}_i$)
- Rotational velocity ($\hat{\omega}_i$)

Translational Velocity ($\hat{v}_i$)

- $M_i \hat{v}_i = \int_{R_i} \rho_i \hat{u} dg_i$
- M : Mass of rigid body
- ρ_i : Density of rigid body
- dg_i : Volume of grid cell occupied by rigid body

Rotational Velocity ($\hat{\omega}_i$)

- $I_i \hat{\omega}_i = \int_{R_i} r_i \times \rho_i \hat{u} dg_i$
- r_i : Vector from grid cell to center of mass of rigid body.
- I_i : Moment of inertia of rigid body

Rigid Body Velocity

- $\hat{u}_R = \cup(\hat{v}_i + \hat{\omega}_i \times r_i)$
- $\hat{u}_R$ is the union of rigid body velocities over all rigid bodies.

Final Velocity

- Distribute rigid body velocity over bodies to get final velocity.
- $u^{n+1} = (1 - w)\hat{u} + w\hat{u}_R$
- w : Fraction of computational cell occupied by rigid body.

Kickoff

- Jos Stam's code for Real-Time Fluid Dynamics For Games.
- Simple and rapid implementation of fluid dynamics solver.
- Added support for rigid bodies and forces.

Additions

- Rigid body objects.
- Interface to add rigid bodies.
- Rigid/Fluid domain grid R/F
- Rigid body velocity vectors grid.

Calculations

- Calculate translational ($\hat{v}_i$) and rotational ($\hat{\omega}_i$) velocities for each body.
- Calculate rigid body velocity ($\hat{u}_R$) for each body.
- Input ($\hat{u}_R$) and current position of rigid body to a rigid body solver to get the next position.
- Move rigid body to next position.

Rigid Body Solver

- A simple rigid body solver implemented for the project.
- Works with square bodies of variable size.
- Translational velocities are totalled and applied on the body's center of mass.
- Rotational velocities are calculated but ignored.
- Rigid bodies are moved over integral distances (cells).
  - Fractional velocities are left over at each time step.
  - Leftover velocities are accrued to use in later time steps.
- Bodies restricted to the boundaries of the grid.

Demo

Conclusion

- Rigid body interaction with fluids achieved in real time and with stability.
- Rigid body collisions not solved in current implementation.

Thank You!